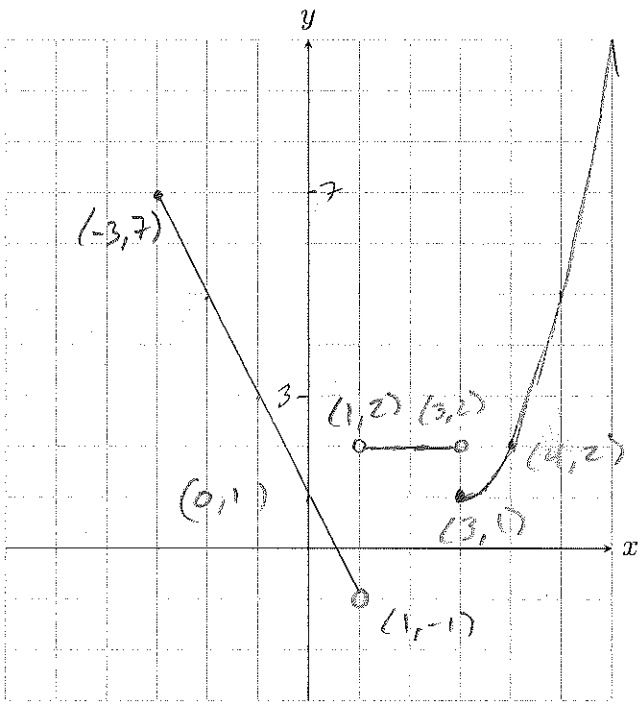
Math 111 WS 5 Piecewise Defined Functions

Name: Solutions

1. A function f defined as

$$f(x) = \begin{cases} -2x+1 & \text{if } -3 \le x < 1 \\ 2 & \text{if } 1 < x < 3 \\ (x-3)^2+1 & \text{if } x \ge 3 \end{cases}$$

a. Graph $y = f(x)$.



b. Find $f(-2)$, $f(1)$, and $f(2)$.

$f(-2) = -2(-2)+1 = 5$

$f(1)$ is undefined

$f(2) = 2$

c. Locate any intercepts.

$-2x+1 = 0$

$x = 1/2$

y-int $= (0,1)$

x-int $= (1/2, 0)$

d. Determine the domain and range of f.

$D = [-3,1) \cup (1,\infty)$

$R = (-1,\infty)$

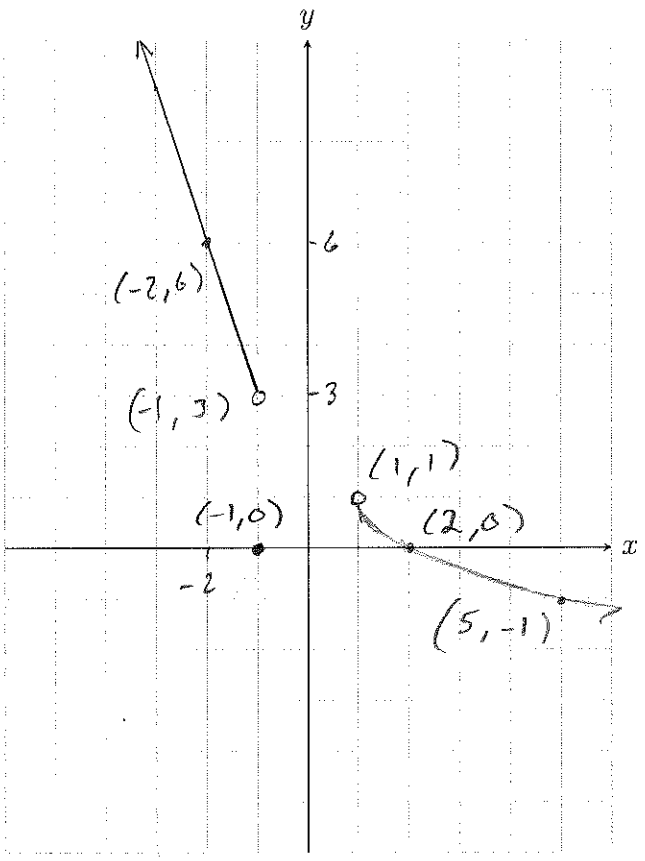
e. Is f continuous *on its domain*?

No. There is a discontinuity at 3, which is in the domain

2. A function f defined as

$$f(x) = \begin{cases} -3x & \text{if } x < -1 \\ 0 & \text{if } x = -1 \\ -\sqrt{x-1}+1 & \text{if } x > 1 \end{cases}$$

a. Graph $y = f(x)$.



b. Find $f(-2)$, $f(-1)$, and $f(0)$.

$f(-2) = 6$

$f(-1) = 0$

$f(0)$ is undefined

c. Locate any intercepts.

x-ints @ $(-1,0)$ & $(2,0)$

d. Determine the domain and range of f.

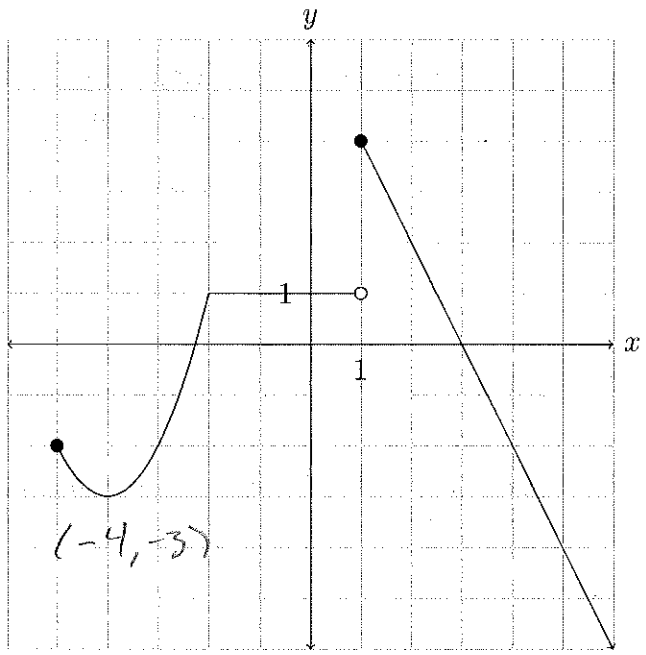
$D = (-\infty, -1] \cup (1, \infty)$

$R = (-\infty, 1) \cup (3, \infty)$

e. Is f continuous *on its domain*?

No, there's a jump at $x=-1$ which is in the domain

3. The graph of a piecewise-defined function is given. Write a definition for the function.



Parabola Part: vertex @ $(-4,3)$, facing upward, no stretch

$y=(x+4)^2-3$ on $[-5,-2)$

Constant: $y=1$ on $[-2,1)$

Linear Part: slope of -2, point of $(1,4)$

$y=m(x-x_1)+y_1$

$=-2(x-1)+4$

$=-2x+6$

$$f(x)=\begin{cases}(x+4)^2-3 & -5\le x<-2\\ 1 & -2\le x<1\\ -2x+6 & x\ge 1\end{cases}$$

4. In the summer of 2009, Duke Energy supplied electricity to residences of Ohio for a monthly customer charge of \$4.50 plus 4.2345 cents per kilowatt-hour (kWhr) for the first 1000 kWhr supplied in the month and 5.3622 cents per kWhr for all usage over 1000 kWhr in the month.

a. What is the charge for using 300 kWhr in a month?

$4.50 + 0.042345 \cdot 300$

$= 17.2035$

It will cost \$17.20 for 300 kWhr.

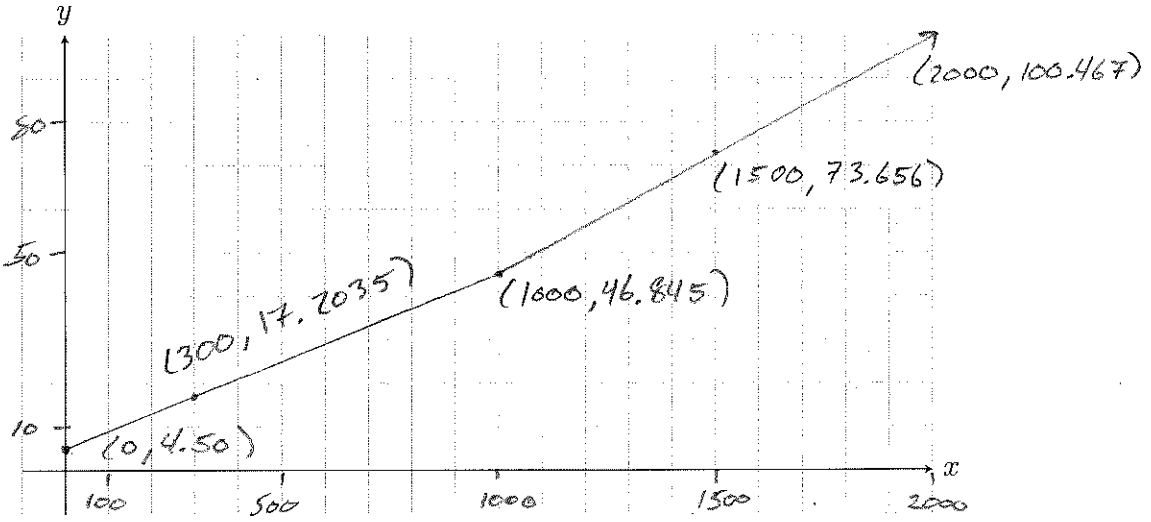
b. What is the charge for using 1500 kWhr in a month?

$4.50 + 0.042345 \cdot 1000 + 0.053622 \cdot 500$

$= 73.656$

It will cost \$73.66 for 1500 kWhr

c. If C is the monthly charge for x kWhr, develop a model relating the monthly charge and kilowatt-hours used. That is, express C as a function of x. Begin by graphing $C = C(x)$.



It switches at 1000 kWhr so $4.50 + 0.042345 \cdot 1000 = 46.845$

at 2000 kWhr: $4.50 + 0.042345 \cdot 1000 + 0.053622 \cdot 1000 = 100.467$

Part 1: $m = 0.042345$, $b = 4.50$ $\rightarrow$ $y = 0.042345x + 4.50$

Part 2: $m = 0.053622$, Point $= (1000, 46.845)$ $\rightarrow$ $y = 0.053622(x - 1000) + 46.845$

$$C(x) = \begin{cases} 0.042345x + 4.50 & 0 \le x \le 1000 \\ 0.053622(x - 1000) + 46.845 & x > 1000 \end{cases}$$