Math 95 WS 7 Radical Functions Name: ________________

§7.1 – 7.5

Chapter 7 is all about radical functions. That is, a function such as $f(x) = 3\sqrt{4x-7}+1$ or $g(x) = -2\sqrt[4]{x+2}$.

To start let's look at a few lists of numbers that you need to memorize:

$1^2 = 1$	$1^3 = 1$	$1^4 = 1$	$1^5 = 1$
$2^2 = 4$	$2^3 = 8$	$2^4 = 16$	$2^5 = 32$
$3^2 = 9$	$3^3 = 27$	$3^4 = 81$	
$4^2 = 16$	$4^3 = 64$	$4^4 = 256$	
$5^2 = 25$	$5^3 = 125$	$5^4 = 625$	
$6^2 = 36$			
$7^2 = 49$			
$8^2 = 64$			
$9^2 = 81$			
$10^2 = 100$			
$11^2 = 121$			
$12^2 = 144$			
$13^2 = 169$			
$14^2 = 196$			
$15^2 = 225$			
$16^2 = 256$			

Why is this important? Well, if I ask you to simplify $\sqrt[5]{32}$ it is more than a little handy to know that $2^5 = 32$.

Okay, now we need to refresh what an n*th* root is.

The n*th root* of a is the number b such that $b^n = a$. Our notation is $\sqrt[n]{a} = b$. Thus $nrt(x) = \sqrt[n]{x}$ is a predefined function which answers the question "what number to the n*th* power equals x?" Let's use this to answer the following:

1. $\sqrt[4]{16} =$

2. $\sqrt[5]{-32} =$

3. $\sqrt[4]{-81} =$

Notice that the last problem has an imaginary answer. Why is this?

Let's do a quick rehash of our basic exponent properties before we continue:

Let p and q be rational numbers written in lowest terms. The following properties hold:
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a. $a^p \cdot a^q = a^{p+q}$

b. $a^{-p} = \frac{1}{a^p}$, $\frac{1}{a^{-p}} = a^p$

c. $\left(\frac{a}{b}\right)^{-p} = \left(\frac{b}{a}\right)^{p}$

d. $\frac{a^p}{a^q} = a^{p-q}$

e. $(a^p)^q = a^{pq}$

f. $(ab)^p = a^p b^p$

g. $\left(\frac{a}{b}\right)^{p} = \frac{a^p}{b^p}$

Here are a few basic problems to refresh these concepts:

4. $x^3 \cdot x^5 =$

5. $x^{-5} =$

6. $\frac{1}{x^{-4}} =$

7. $\left(\frac{3}{x}\right)^{-2} =$

8. $\frac{x^5}{x^3} =$

9. $(x^2)^5 =$

10. $\left(\frac{x^3}{4}\right)^2 =$

11. $\left(\frac{3x^5y^{-1}}{x^2}\right)^{-3} =$

Let's now consider the equation $(5^x)^2 = 5$. Using (e.) from above we see that we may rewrite this equation as $5^{2x} = 5^1$. Does this help us see what x should be to make the $=$ sign true?

So we see that $(5^{1/2})^2 = 5$ but isn't this the same as writing $5^{1/2} = \sqrt{5}$?

Can we generalize this? Of course! Let's look at the equation $(a^x)^n = a$. Again, we rewrite this as $a^{nx} = a^1$ so we see x must equal $1/n$ so $(a^{1/n})^n = a$. And again we rewrite this in the form $a^{1/n} = \sqrt[n]{a}$. Let's take this one step further. Let's take both sides of the last equation and raise them to the mth power:

$$(a^{1/n})^m = (\sqrt[n]{a})^m$$
$$\Rightarrow a^{m/n} = \sqrt[n]{a^m}$$

Let's highlight that last part:

$$a^{m/n} = \sqrt[n]{a^m}$$

Use this fact to simplify the following:

12. $64^{-1/3} =$

13. $81^{-3/4} =$

14. $\sqrt{x} \cdot \sqrt[3]{x} =$

15. $\sqrt[3]{27x^2} =$

16. $\frac{\sqrt[4]{16x}}{\sqrt[3]{x}} =$

17. $\left(\frac{x^2}{81}\right)^{-1/2} =$

18. $\sqrt[3]{\sqrt{x+1}} =$

19. $\sqrt[5]{c^15} =$

20. $\frac{y^{-1/2}}{x^{-1/3}} =$

21. $sqrtx(\sqrt{x} - 1) =$

There's one other little tidbit that we should review before moving on to more complicated expressions:

Consider $\sqrt{3^2} = 3$, $\sqrt{(-4)^2} = \sqrt{16} = 4$ and $\sqrt{(-6)^2} = 6$. What can we deduce using this information about the value of $\sqrt{x^2}$ in general?

Use the answer to the above question to answer the following:

22. $\sqrt{(-3)^2}$

23. $\sqrt{(x+1)^2}$

24. $\sqrt{z^2-4z+4}$

Take procedure (f.) and (g.) from above and let's see how this translates to nth roots:

By (f.) $a^p \cdot b^p = (ab)^p$. If $p = {}^1\!/\!_n$ then we get $\sqrt[n]{a} \cdot \sqrt[n]{b} = a^{1/n} \cdot b^{1/n} = (ab)^{1/n} = \sqrt[n]{ab}$. Or, in short, $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$.

By (g.) $\left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}$. If $p = {}^1\!/\!_n$ then we get $\sqrt[n]{\frac{a}{b}} = \left(\frac{a}{b}\right)^{1/n} = \frac{a^{1/n}}{b^{1/n}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$. Or, in short, $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$.

Let's highlight the final results:

$$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$$

$$\sqrt[n]{\tfrac{a}{b}} = \tfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

We now want to use these results to simplify expressions. Say you want to simplify $\sqrt[4]{256x^7y^{12}z^9}$, then we use the first tool above to get:

$$\begin{aligned}
\sqrt[4]{256x^7y^{12}z^9} &= \sqrt[4]{256} \cdot \sqrt[4]{x^4} \cdot \sqrt[4]{x^3} \cdot \sqrt[4]{y^4}\sqrt[4]{y^4}\sqrt[4]{y^4}\sqrt[4]{z^4}\sqrt[4]{z^4}\sqrt[4]{z} \\
&= 4 \cdot x \cdot \sqrt[4]{x^3} \cdot y \cdot y \cdot y \cdot z \cdot z \cdot \sqrt[4]{z} \\
&= 4xy^3z^2\sqrt[4]{x^3}\sqrt[4]{z} \\
&= 4xy^3z^2\sqrt[4]{x^3z}
\end{aligned}$$

We can actually do this faster. Redo this problem as I show it on the board with explanations for each step to the right.

Let's now look at an example that uses the second tool above. Say you want to simplify $\sqrt[3]{\frac{54x^6y^7}{z^6}}$, then we use both the first *and* second tools to realize that we can just take the 3rd root of each factor on top and bottom, leaving anything we cannot take the 3rd root of inside the radical.

$$\sqrt[3]{\frac{54x^6y^7}{z^6}} = \frac{3x^2y^2\sqrt[3]{2y}}{z^2}$$

We can also use the tools in the following manner:

$$\sqrt{\frac{6x^3}{8z}} \cdot \sqrt{\frac{3x^6}{2z^3}} = \sqrt{\frac{36x^9}{16z^4}}$$
$$= \frac{6x^4\sqrt{x}}{4z^2}$$

Now it's your turn to try!

25. $\sqrt{5} \cdot \sqrt{20} =$

26. $\sqrt[4]{\frac{1}{3}} \cdot \sqrt[4]{\frac{1}{9}} \cdot \sqrt[4]{\frac{1}{3}} =$

27. $\sqrt[3]{2a} \cdot \sqrt[3]{5a} =$

28. $\sqrt[5]{\frac{2x}{y}} \cdot \sqrt[5]{\frac{16y}{x}} =$

29. $\sqrt{300} =$

30. $\sqrt[4]{512} =$

31. $\sqrt{25x^4} =$

32. $\sqrt[3]{-16x^3y^5} =$

33. $\sqrt[3]{2a} \cdot \sqrt[3]{4a^2b} =$

34. $\sqrt{5} \cdot \sqrt[4]{5} =$

35. $\sqrt[3]{x}\sqrt[4]{x} =$

36. $\sqrt[3]{\frac{5}{8}} =$

37. $\sqrt{\frac{16}{y^2}} =$

38. $\frac{\sqrt{40}}{\sqrt{10}} =$

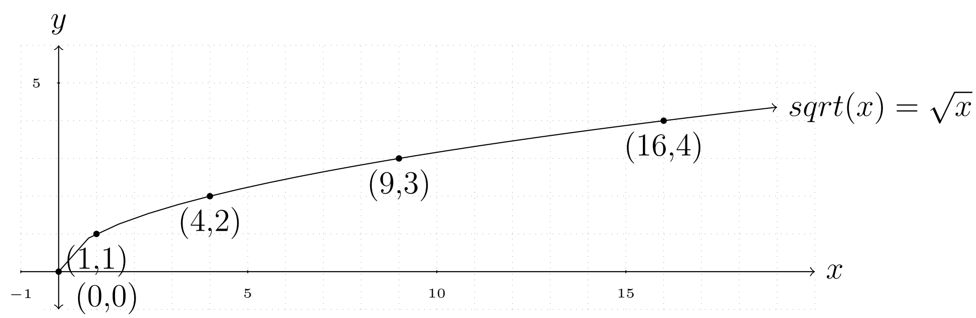
39. $\frac{\sqrt{x^2y}}{\sqrt{y}} =$

40. $\sqrt[4]{\frac{16x^3}{y^4}} =$

41. $\sqrt{\frac{5a^2}{8}} \cdot \sqrt{\frac{5a^3}{2}} =$

We now turn to graphing radical functions. Let's begin by graphing the simplest radical function, $sqrtx = \sqrt{x}$. To do this, we begin by making a table of values and then plot the points.

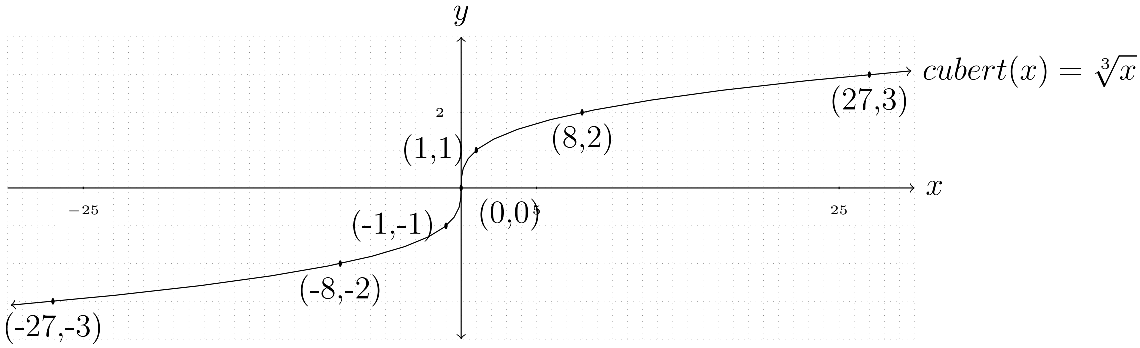
x	$sqrt(x)$
0	0
1	1
4	2
9	3
16	4



Why didn't I choose any x values less than zero?

Let's also get one other basic radical function graphed, $cubert(x) = \sqrt[3]{x}$. In this case we get the following table of values and graph:

x	$cubert(x)$
-27	-3
-8	-2
-1	-1
0	0
1	1
8	2
27	3



We are now going to see how we may *flip*, *stretch*, *compress*, and *translate* both the $cubert(x)$ and $sqrt(x)$ to get the graphs for other cuberoot and squareroot functions.

What does the word *translate* mean?

When I say *stretch* or *compress*, how specifically am I stretching or compressing?

To start let's look at flipping. If I were to reflect $sqrt(x)$ over the x-axis, how have the outputs of my function changed?

Complete the following table then graph $f(x) = -\sqrt{x}$ in the space to the right.

x	$f(x) = -\sqrt{x}$
-2	
-1	
0	
1	
2	

So, a negative outside of $\sqrt{x}$ takes each y value and makes them negative, thus resulting in a square root graph which has been reflected over the x-axis.

Next let's look at stretches and compressions. Begin by filling out the following two tables and graphing the functions in the space to the right.

x	$f(x) = 2\sqrt[3]{x}$
-8	
-1	
0	
1	
8	

x	$f(x) = \frac{1}{4}\sqrt{x}2$
0	
1	
4	
9	
16	

What have we witnessed here?

So, when we have $f(x) = a\sqrt{x}$ or $g(x) = a\sqrt[3]{x}$ the a value will simply multiply by each y value from $sqrt(x) = \sqrt{x}$ or $cubert(x) = \sqrt[3]{x}$ respectively, resulting in a stretch or compression and possibly a flip (if a is negative).

Next we need to look at translations. We'll start with the easy ones. Fill out the following tables and graph the function in the space to the right.

x	$f(x) = \sqrt[3]{x} + 3$
-8	
-1	
0	
1	
8	

x	$f(x) = \sqrt{x} - 1$
0	
1	
4	
9	
16	

So here we see the function being shifted up and down since we are simply adding or subtracting to the y values of the basic function.

Now the hard part. These tables are going to be a bit more difficult, but work in groups and see if you can figure out how to complete them. And, of course, graph them to the right.

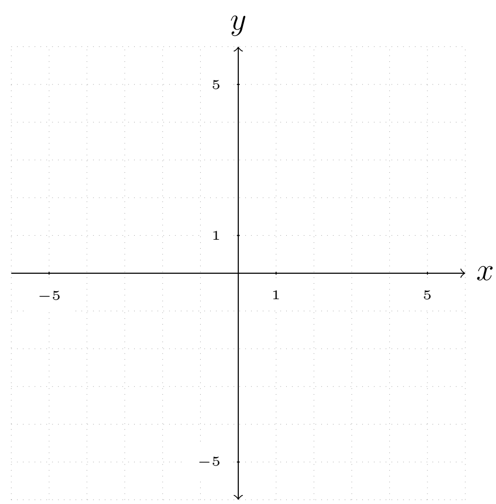
x	$f(x) = \sqrt{x - 2}$
	0
	1
	2
	3
	4

x	$f(x) = \sqrt[3]{x+1}$
	-2
	-1
	0
	1
	2

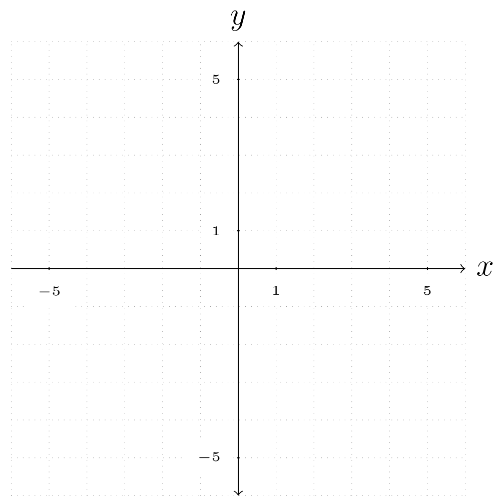
The key to completing these tables is to ask, "what does x need to be in order for y to be ____?" For the second example above what we see is that x needs to be 1 less than it normally would in order to get the same outputs we would in $\sqrt[3]{x+1}$. What this results in is a shift to the left 1 unit. Remember we think of horizontal shifts as *counter*-intuitive.

Let's put these all together with the following examples. For the next two problems identify the transformations and then use them to graph the resultant parabola.

42. $f(x) = \sqrt{x-1} + 2$



43. $f(x) = 2\sqrt[3]{x+2} - 3$



Alright, now that we have a grasp on how to graph a square root or cube root type of function, let's look at the domain of these kinds of functions. To start, let's remind ourselves what the definition of domain is:

So if we have a root with *even* index, then we need to consider the *radicand*. What is the radicand and why do we only care about this when discussing radicals of even index?

So, if $f(x) = \sqrt[4]{3x-2}$, the domain is the set of x values such that $3x - 2 \geq 0$. In this case it works out to be $D = \{x | x \geq \frac{2}{3}\}$.

Determine the domain of the following functions.

44. $f(x) = \sqrt{3x-4}$

45. $f(x) = \sqrt[3]{2x^2-5}$

46. $f(x) = \sqrt[6]{2x^2-4}$

47. $f(x) = \sqrt{x^2+1}$

48. $f(x) = \frac{1}{\sqrt{9x-7}}$

We now reach the last thing we will cover this term! We'll be finishing up by going over how to solve equations involving radical expressions. Here, I'll give away the punch line right now: Start by solving for the radical, then square or cube or take both sides to the fourth power or whatever power will eliminate the radical. You should then be in a familiar situation - well, hopefully. Let's take a look:

Say I want to solve $\sqrt{4-x}+5=8$. Copy down how I solve this on the board here.

We may end up in a bit more complex of a situation such as needing to solve $\sqrt{3x}+2=\sqrt{x-1}$. Copy down my technique here.

What if we need to solve $\sqrt[4]{x+2}-27=54$?

What is the general idea of solving an equation?

So if I have the equation $x^{3/4}=27$, what might our strategy be when solving for x?

Suppose we have $3(x+4)^3+7=31$. Copy down how I solve this here.

Here's some practice for you. Solve the following equations for x.

49. $\sqrt{2x-1} = 3$

50. $sqrt3x + 3 = 3x - 1$

51. $sqrt2x - 1 = \sqrt{x+1}$

52. $\sqrt[3]{4x-7} = 4$

53. $x^3 = -64$

54. $2(x-1)^4 = 32$

55. $x^{2/3} = 25$

As a quick final discussion, let's review how to solve equations graphically. Suppose $x^{2/5} = 4 - x^3$. Use your calculator to solve for x by graphing. To do this we set $y_1(x) = x^{2/5}$ and $y_2(x) = 4 - x^3$, graph them both in an appropriate viewing window and then use the calculate feature to determine where the intersections are. Remember, when I use the word "where" I mean, "for what x-values do the graphs hold the same y-values?" Why is it that the x-values of the intersections of the graphs give us the solution to the equation? What is the solution to the given equation?

Use your calculator to solve the following equations. Write your solutions in set notation.

56. $x^{3/5} + 1 = 2 - x^2$

57. $\sqrt[3]{x+1} = x^2 - x^{2/3}$