Math 111 WS 7 Inverse Functions

Name: Solutions

1. Given the following functions, what are their inverses? What are the domains and ranges of each?

a. $\{(-2,6),(-1,3),(0,2),(1,5),(2,8)\}$

Inverse: $\{(6,-2),(3,-1),(2,0),(5,1),(8,2)\}$

$D_{orig} = \{-2,-1,0,1,2\}$

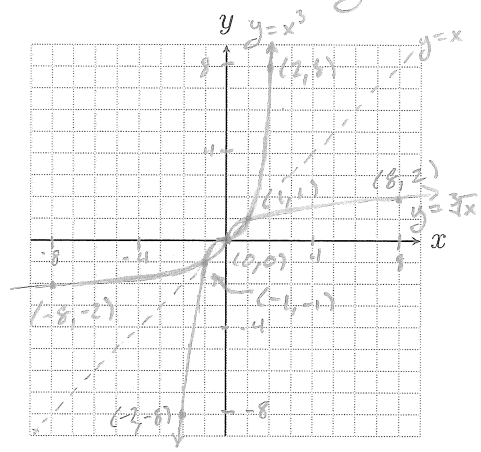
$R_{orig} = \{6,3,2,5,8\}$

$D_{inv} = \{6,3,2,5,8\}$

$R_{inv} = \{-2,-1,0,1,2\}$

b. $g(x) = x^3$

$g^{-1}(x) = \sqrt[3]{x}$



$D_g = \mathbb{R}$

$R_g = \mathbb{R}$

$D_{g^{-1}} = \mathbb{R}$

$R_{g^{-1}} = \mathbb{R}$

c. $f(x) = 2x+3$

$y = 2x+3$

$x = 2y+3$

$x-3 = 2y$

$y = \frac{1}{2}x - \frac{3}{2}$

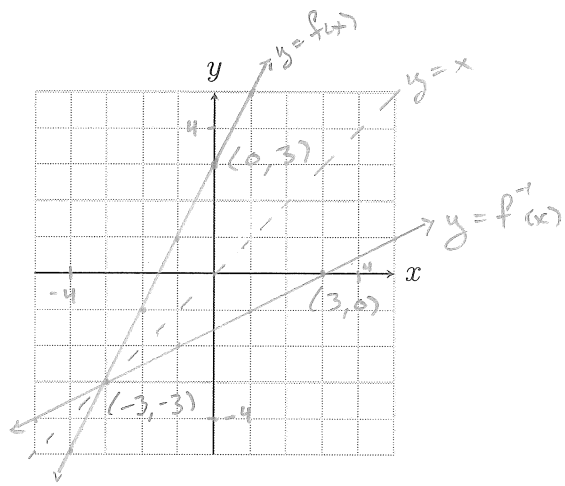
$f^{-1}(x) = \frac{1}{2}x - \frac{3}{2}$

$D_f = \mathbb{R}$

$R_f = \mathbb{R}$

$D_{f^{-1}} = \mathbb{R}$

$R_{f^{-1}} = \mathbb{R}$



d. The function $F(C) = \frac{9}{5}C + 32$, which converts temperature from C degrees Celsius to F degrees Fahrenheit.

$F = \frac{9}{5}C + 32$

$F - 32 = \frac{9}{5}C$

$C = \frac{5}{9}(F-32)$

$C(F) = \frac{5}{9}(F-32)$

$F^{-1} = C$

$D_F = \mathbb{R}$ $D_C = \mathbb{R}$

$R_F = \mathbb{R}$ $R_C = \mathbb{R}$

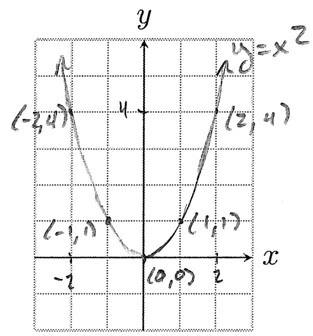
Here I'm not switching the input + output for the sake of retaining their definitions of Fahrenheit + Celsius.

2. The following two functions are NOT invertible. Explain why not.

a. For the following function, the domain represents the age of five males and the range represents their HDL (good) cholesterol (mg/dL).

Age	HDL Cholesterol
38	57
42	54
46	34
55	38
61	38

b. $f(x) = x^2$



If we swith the inputs & outputs, the new relationship wouldn't be a function as some inputs would have multiple outputs.

3. What is the definition of a **one-to-one** function?

A function where each output only has one input. Formally: If $f(x_1)=y$ & $f(x_2)=y$, then $x_1 = x_2$

4. What is the definition of an **inverse** function?

Given a function f, its invers function, f^{-1}, takes the outputs of f as inputs, & then outputs the associated input of f. That is, it goes backwards!

5. Determine whether the following functions are inverses of each other.

a. $f(x) = \frac{5}{2} - \frac{1}{2}x$ and $g(x) = -2x + 5$

$$f(g(x)) = \frac{5}{2} - \frac{1}{2}(-2x+5)$$
$$= \frac{5}{2} + x - \frac{5}{2}$$
$$= x \quad \checkmark$$

$$g(f(x)) = -2\left(\frac{5}{2} - \frac{1}{2}x\right) + 5$$
$$= -5 + x + 5$$
$$= x \quad \checkmark$$

Yes, f & g are inverses.

b. $f(x) = -\frac{6}{x-1}$ and $g(x) = \frac{6+x}{x}$

$$f(g(x)) = -\frac{6}{\left(\frac{6+x}{x}\right) - 1} \cdot \frac{x}{x}$$
$$= -\frac{6x}{6+x-x}$$
$$= -\frac{6x}{6} = -x$$

f & g are <u>not</u> inverses

6. Find the inverse of the following functions and then state the domain and range of each.

a. $f(x) = \dfrac{2x+1}{x-1}$

$y = \dfrac{2x+1}{x-1}$ $\qquad D_f = \{x \mid x \neq 1\}$

$x = \dfrac{2y+1}{y-1}$ $\qquad R_f = \{y \mid y \neq 2\}$

$x(y-1) = 2y+1$

$xy - x = 2y + 1$

$xy - 2y = 1 + x$

$y(x-2) = 1+x$ $\qquad D_{f^{-1}} = \{x \mid x \neq 2\}$

$y = \dfrac{1+x}{x-2}$ $\qquad R_{f^{-1}} = \{y \mid y \neq 1\}$

$f^{-1}(x) = \dfrac{1+x}{x-2}$

b. $g(x) = \dfrac{x-5}{3x+2}$

$D_g = \{x \mid x \neq \frac{-2}{3}\}$

$R_g = \{y \mid y \neq \frac{1}{3}\}$

$y = \dfrac{x-5}{3x+2}$

$x = \dfrac{y-5}{3y+2}$

$3xy + 2x = y - 5$

$3xy - y = -2x - 5$

$y(3x-1) = -2x - 5$

$y = \dfrac{-2x-5}{3x-1}$ $\qquad D_{g^{-1}} = \{x \mid x \neq \frac{1}{3}\}$

$f^{-1}(x) = \dfrac{-2x-5}{3x-1}$ $\qquad R_{g^{-1}} = \{y \mid y \neq \frac{-2}{3}\}$

7. Restrict the domain of $f(x) = (x-2)^2 + 1$ so that it is one-to-one, find its inverse, state the domain and range of both, and graph both on the same axes.

Let $D_f = [2, \infty)$ $\qquad R_f = [1, \infty)$

$y = (x-2)^2 + 1$

$x = (y-2)^2 + 1$

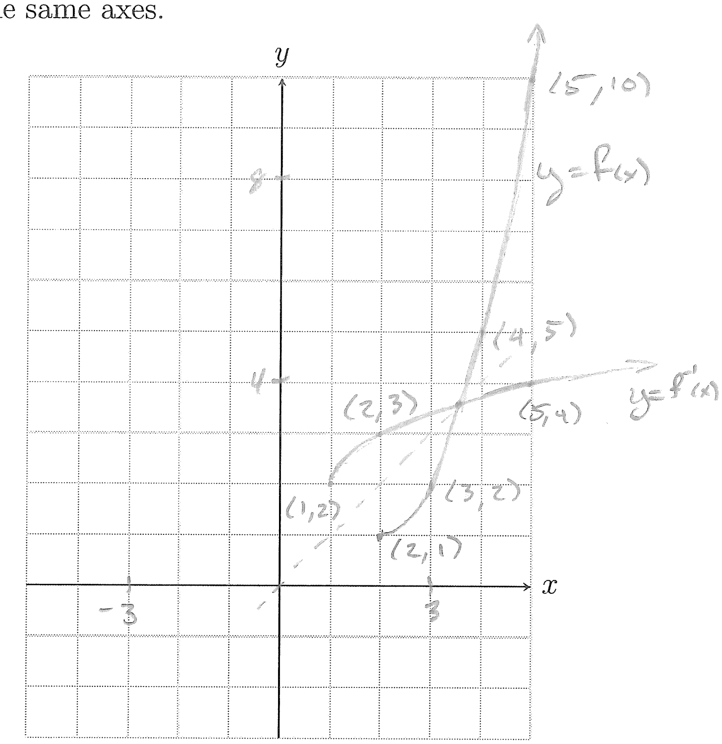
$x - 1 = (y-2)^2$

$\sqrt{x-1} = y - 2$

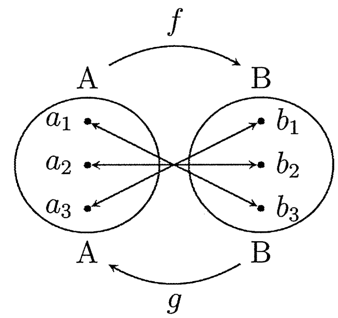
$y = \sqrt{x-1} + 2$

$f^{-1}(x) = \sqrt{x-1} + 2$ $\qquad D_{f^{-1}} = [1, \infty)$

$R_{f^{-1}} = [2, \infty)$



Math 111 Invertible Activity



Starting questions:

1. What is the requirement for a relationship between inputs and outputs to be considered a function?

2. What requirement (that we deal with in this class) do we need in order for a function to be invertible?

Step 1: Come up with 3 or 4 functions which are **not** invertible. State the domain and range (image) of each function. Be creative! You may describe your functions using a graph, symbolically, using an internal diagram, a table, a set of ordered pairs, or with words. Can you come up with a non-invertible function using different methods of description?!

Step 2: In your groups discuss all of the functions you've come up with and determine if they all satisfy being **not** invertible. Discuss why each is or is not invertible.

Step 3: Choose two functions from the group and determine how to "fix" them so that the functions **are** invertible. Is it appropriate to say that these are the same functions as what you started with? Discuss what you did in order to fix them. Each student should write the original non-invertible function, the "fixed" version of the function, and the inverse of the fixed version on their own paper.