Math 111 WS 9, Logarithmic Functions Introduction Name: Solutions

1. Change each exponential statement to an equivalent statement involving a logarithm.

a. $1.2^3 = m$

$\log_{1.2} m = 3$

b. $e^b = 9$

$\log_e 9 = b$

$\ln 9 = b$

c. $a^4 = 24$

$\log_a 24 = 4$

2. Change each logarithmic statement into an equivalent statement involving an exponent.

a. $\log_a(4) = 5$

$a^5 = 4$

b. $\ln(b) = -3$

$e^{-3} = b$

c. $\log_3(5) = c$

$3^c = 5$

3. Evaluate the following:

a. $\log_2(16) = 4$

$2^? = 16$

b. $\log_3\left(\frac{1}{27}\right) = -3$

$3^? = \frac{1}{27}$

c. $\log_{\frac{1}{2}}(8) = -3$

$\left(\frac{1}{2}\right)^? = 8$

d. $\log_5(1) = 0$

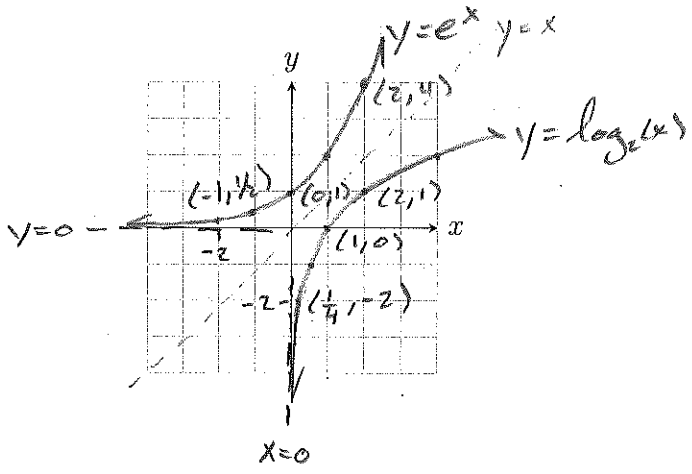
$5^? = 1$

4. Graph $exp_2(x) = 2^x$ and $y = \log_2(x)$ on the same set of axes. State the domain and range of both.

$y = 2^x$

x	y
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4

$y = \log_2 x$ y x

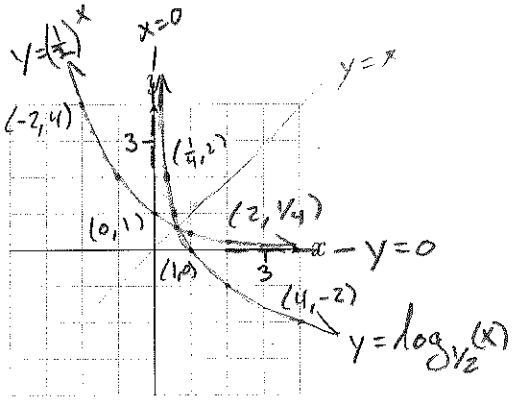


5. Graph $exp_{\frac{1}{2}}(x) = \left(\frac{1}{2}\right)^x$ and $y = \log_{\frac{1}{2}}(x)$ on the same set of axes. State the domain and range of both.

$y = (\frac{1}{2})^x$

x	y
-2	4
-1	2
0	1
1	1/2
2	1/4

$y = \log_{\frac{1}{2}} x$ y x

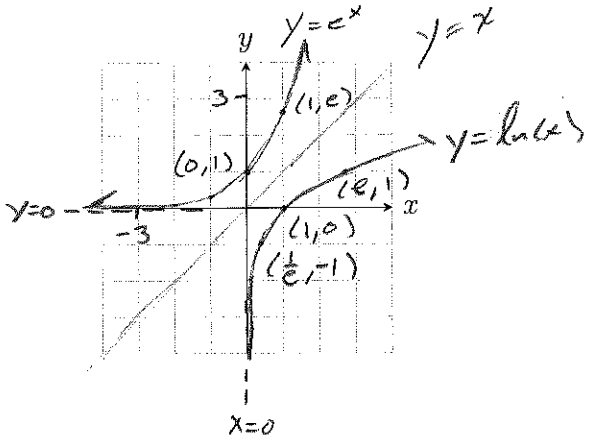


6. Graph $f(x) = e^x$ and $y = \ln(x)$ on the same set of axes. State the domain and range of both.

$y = e^x$

x	y
-2	$e^{-2} \approx 0.1$
0	1
1	$e \approx 2.72$
2	$e^2 \approx 7.4$

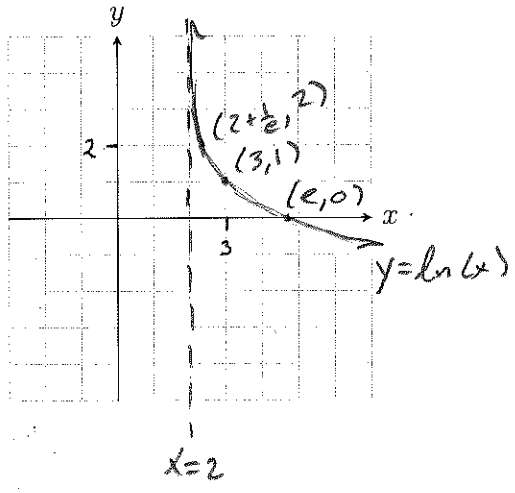
$y = \ln(x)$ y x



7. Use transformations to graph $f(x) = -\ln(x-2) + 1$ then state the domain and range of f.

Key stuff from $y = \ln(x)$

	negate ys	up 1	right 2
x=0	x=0	x=0	x=2
(e, 1)	(e, -1)	(e, 0)	(e+2, 0)
(1, 0)	(1, 0)	(1, 1)	(3, 1)
(1/e, -1)	(1/e, 1)	(1/e, 2)	(2+1/e, 2)



8. Consider $f(x) = 3\log(x-1)$. Find f^{-1}, graph f and f^{-1} on the same coordinate plane, and state the domain and range of both.

$y = 3\log(x-1)$

$\frac{y}{3} = \log(x-1)$

$10^{y/3} = x - 1$

$x = 10^{y/3} + 1$

$f^{-1}(x) = 10^{\frac{1}{3}x} + 1$

Key stuff on $y = \log(x)$: $(\frac{1}{10}, -1)$, $(1, 0)$, $(10, 1)$, $x = 0$

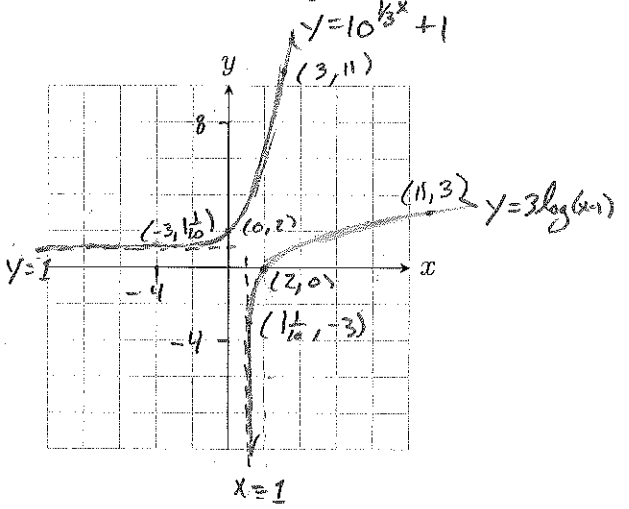
mult. ys by 3: $(\frac{1}{10}, -3)$, $(1, 0)$, $(10, 3)$, $x = 0$

right 1: $(1\frac{1}{10}, -3)$, $(2, 0)$, $(11, 3)$, $x = 1$

$y = 10^x$: $(-1, 1/10)$, $(0, 1)$, $(1, 10)$, $y = 0$

up 1: $(-1, 1\frac{1}{10})$, $(0, 2)$, $(1, 11)$, $y = 1$

mult. xs by 3: $(-3, 1\frac{1}{10})$, $(0, 2)$, $(3, 11)$, $y = 1$



9. Find the domain of each logarithmic function.

a. $F(x) = \log_2(x+3)$

$x + 3 > 0$

$x > -3$

$D = (-3, \infty)$

b. $g(x) = \log_5\left(\left|\frac{1+x}{1-x}\right|\right)$

$1 + x \neq 0$

$D = \{x \mid x \neq -1\}$.

c. $h(x) = \log_{\frac{1}{2}}(|x|)$

$D = \{x \mid x \neq 0\}$

10. Solve the following equations.

a. $\log_3(4x-7) = 2$

$3^2 = 4x - 7$

$16 = 4x$

$x = 4$

The sol set is $\{4\}$.

b. $e^{2x} = 5$

$2x = \ln(5)$

$x = \frac{\ln(5)}{2}$

The sol set is $\{\frac{\ln(5)}{2}\}$.

c. $2 \cdot 10^{2-x} = 5$

$10^{2-x} = 5/2$

$2 - x = \log(5/2)$

$2 - \log(5/2) = x$

The sol set is $\{2 - \log(5/2)\}$.

d. $\log_2(8^x) = -3$

$2^{-3} = 8^x$

$2^{-3} = 2^{3x}$

$x = -1$

The sol set is $\{-1\}$.

e. $\ln e^x = 5$

$e^x = e^5$

The solution set is $\{5\}$.

f. $\ln(3x - 2) = 5$

$3x - 2 = e^5$

$3x = e^5 + 2$

$x = \frac{e^5 + 2}{3}$

The sol set is $\left\{\frac{e^5 + 2}{3}\right\}$.

g. $4e^{x+1} = 5$

$e^{x+1} = 5/4$

$x + 1 = \ln(5/4)$

$x = \ln(5/4) - 1$

The sol set is $\{\ln(5/4) - 1\}$.

h. $8 \cdot 3^{2x-7} = 4$

$3^{2x-7} = 1/2$

$2x - 7 = \log_3(1/2)$

$2x = \log_3(1/2) + 7$

$x = \frac{\log_3(1/2) + 7}{2}$

The sol set is $\left\{\frac{\log_3(1/2) + 7}{2}\right\}$.